Modern Physics for Scientists and Engineers

Class Exercise IV

Momentum

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Conservation of Momentum and Energy

**Problem 1:**

Two blocks of masses $M$ and $3M$ are placed on a horizontal, frictionless surface. A light spring is attached to one of them, and the blocks are pushed together with the spring between them (Fig. P9.4). A cord initially holding the blocks together is burned; after this, the block of mass $3M$ moves to the right with a speed of 2.00 m/s. (a) What is the speed of the block of mass $M$? (b) Find the original elastic potential energy in the spring if $M = 0.350$ kg.

**Figure P9.4**
Conservation of Momentum and Energy

The solution

(a) For the system of two blocks \( \Delta p = 0 \),

or \( p_i = p_f \)

Therefore,

\[ 0 = Mv_m + (3M)(2.00 \text{ m/s}) \]

Solving gives \( v_m = -6.00 \text{ m/s} \) (motion toward the left).

(b) \( \frac{1}{2} \frac{k}{M}x^2 = \frac{1}{2} Mv_M^2 + \frac{1}{2} (3M)v_{3M}^2 = 8.40 \text{ J} \)
**Problem 2:**

An estimated force–time curve for a baseball struck by a bat is shown in Figure P9.7. From this curve, determine (a) the impulse delivered to the ball, (b) the average force exerted on the ball, and (c) the peak force exerted on the ball.

![Force-time curve](image)

*Figure P9.7*
Impulse-Momentum Theorem

The solution

(a) \[ I = \int F \, dt = \text{area under curve} \]

\[ I = \frac{1}{2} \left( 1.50 \times 10^{-3} \text{ s} \right) (18000 \text{ N}) = 13.5 \text{ N} \cdot \text{s} \]

(b) \[ F = \frac{13.5 \text{ N} \cdot \text{s}}{1.50 \times 10^{-3} \text{ s}} = 9.00 \text{ kN} \]

(c) From the graph, we see that \( F_{\text{max}} = 18.0 \text{ kN} \)
Elastic Collisions in 1D

Problem 3:

A neutron in a nuclear reactor makes an elastic head-on collision with the nucleus of a carbon atom initially at rest. (a) What fraction of the neutron’s kinetic energy is transferred to the carbon nucleus? (b) If the initial kinetic energy of the neutron is $1.60 \times 10^{-13}$ J, find its final kinetic energy and the kinetic energy of the carbon nucleus after the collision. (The mass of the carbon nucleus is nearly 12.0 times the mass of the neutron.)
Elastic Collisions in 1D

The solution

(a) From the lecture (p 24)

\[ v_{2f} = \left( \frac{2m_1}{m_1 + m_2} \right) v_{1i} \]

\[ f_2 = \frac{E_{2f}^k}{E_{1i}^k} = \frac{1/2m_2v_{2f}^2}{1/2m_1v_{1i}^2} = \frac{m_2}{m_1} \left( \frac{v_{2f}}{v_{1i}} \right)^2 \]

\[ f_2 = \frac{4m_1m_2}{(m_1 + m_2)^2} \]

where \( m_2 \) is the moderator nucleus and in this case, \( m_2 = 12m_1 \)

\[ f_2 = \frac{4m_1(12m_1)}{(13m_1)^2} = \frac{48}{169} = 0.284 \text{ or } 28.4\% \]

of the neutron energy is transferred to the carbon nucleus.

(b) \[ K_C = (0.284)(1.6 \times 10^{-13} \text{ J}) = 4.54 \times 10^{-14} \text{ J} \]

\[ K_n = (0.716)(1.6 \times 10^{-13} \text{ J}) = 1.15 \times 10^{-13} \text{ J} \]
Problem 4:

The mass of the blue puck in Figure P9.31 is 20.0% greater than the mass of the green one. Before colliding, the pucks approach each other with momenta of equal magnitudes and opposite directions, and the green puck has an initial speed of 10.0 m/s. Find the speeds of the pucks after the collision if half the kinetic energy is lost during the collision.
Elastic Collisions in 2D

The initial momentum of the system is 0. Thus,

\[(1.20m)v_{Bi} = m(10.0 \text{ m/s})\]

and \[v_{Bi} = 8.33 \text{ m/s}\]

\[K_i = \frac{1}{2}m(10.0 \text{ m/s})^2 + \frac{1}{2}(1.20m)(8.33 \text{ m/s})^2 = \frac{1}{2}m(183 \text{ m}^2/\text{s}^2)\]

\[K_f = \frac{1}{2}m(v_G)^2 + \frac{1}{2}(1.20m)(v_B)^2 = \frac{1}{2}\left(\frac{1}{2}m(183 \text{ m}^2/\text{s}^2)\right)\]

or \[v_G^2 + 1.20v_B^2 = 91.7 \text{ m}^2/\text{s}^2 \quad (1)\]

From conservation of momentum,

\[mv_G = (1.20m)v_B\]

or \[v_G = 1.20v_B \quad (2)\]

Solving (1) and (2) simultaneously, we find

\[v_G = 7.07 \text{ m/s} \quad \text{(speed of green puck after collision)}\]

and \[v_B = 5.89 \text{ m/s} \quad \text{(speed of blue puck after collision)}\]
Problem 5:

A system consists of three particles located as shown in Figure 9.21a. Find the center of mass of the system.

\[ m_1 = m_2 = 1 \text{ kg} \]
\[ m_3 = 2 \text{ kg} \]
Center-of-Mass

The solution

\[
x_{CM} = \frac{\sum m_i x_i}{M} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}
\]

\[
= \frac{(1.0 \text{ kg})(1.0 \text{ m}) + (1.0 \text{ kg})(2.0 \text{ m}) + (2.0 \text{ kg})(0)}{1.0 \text{ kg} + 1.0 \text{ kg} + 2.0 \text{ kg}}
\]

\[
= \frac{3.0 \text{ kg} \cdot \text{m}}{4.0 \text{ kg}} = 0.75 \text{ m}
\]

\[
y_{CM} = \frac{\sum m_i y_i}{M} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}
\]

\[
= \frac{(1.0 \text{ kg})(0) + (1.0 \text{ kg})(0) + (2.0 \text{ kg})(2.0 \text{ m})}{4.0 \text{ kg}}
\]

\[
= \frac{4.0 \text{ kg} \cdot \text{m}}{4.0 \text{ kg}} = 1.0 \text{ m}
\]
The position vector to the center of mass measured from the origin is therefore

$$\mathbf{r}_{\text{CM}} = x_{\text{CM}} \mathbf{i} + y_{\text{CM}} \mathbf{j} = (0.75 \mathbf{i} + 1.0 \mathbf{j}) \text{ m}$$

We can verify this result graphically by adding together $m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2 + m_3 \mathbf{r}_3$ and dividing the vector sum by $M$, the total mass. This is shown in Figure 9.21b.
Homework

1

Momentum conservation

A 3.00-kg steel ball strikes a wall with a speed of 10.0 m/s at an angle of 60.0° with the surface. It bounces off with the same speed and angle (Fig. P9.9). If the ball is in contact with the wall for 0.200 s, what is the average force exerted by the wall on the ball?
Two blocks are free to slide along the frictionless wooden track $ABC$ shown in Figure P9.20. The block of mass $m_1 = 5.00$ kg is released from $A$. Protruding from its front end is the north pole of a strong magnet, repelling the north pole of an identical magnet embedded in the back end of the block of mass $m_2 = 10.0$ kg, initially at rest. The two blocks never touch. Calculate the maximum height to which $m_1$ rises after the elastic collision.
Two shuffleboard disks of equal mass, one orange and the other yellow, are involved in an elastic, glancing collision. The yellow disk is initially at rest and is struck by the orange disk moving with a speed of 5.00 m/s. After the collision, the orange disk moves along a direction that makes an angle of 37.0° with its initial direction of motion. The velocities of the two disks are perpendicular after the collision. Determine the final speed of each disk.
A water molecule consists of an oxygen atom with two hydrogen atoms bound to it (Fig. P9.39). The angle between the two bonds is 106°. If the bonds are 0.100 nm long, where is the center of mass of the molecule?